PLANETARY ORBITAL DYNAMICS (PLANODYN) SUITE FOR LONG TERM PROPAGATION IN PERTURBED ENVIRONMENT

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ABSTRACT

This article describes the Planetary Orbital Dynamics (PlanODyn) suite for long term propagation in perturbed environment. The dynamical model used for deriving the averaged equation of motion is summarised and the physical models for the spacecraft and planetary environment presented. Some applications of PlanODyn to design of end-of-life disposal are shown; first, the stability analysis and optimisation of re-entry trajectories from highly elliptical orbits. Then, the design of end-of-life re-entry through solar radiation pressure is shown. Two sail strategies are discussed: a passively stabilised solar sails and a method for passive modulation of the sail to decrease the deorbiting time and allowing the deorbiting from Medium Earth Orbits.

Index Terms— Orbit perturbations, averaging, end-of-life trajectory design, solar sail deorbit.

1. INTRODUCTION

Trajectory design and orbit maintenance are a challenging task when multi-body dynamics is involved or in the vicinity of a planet, where the effect of orbit perturbations is relevant. This is the case of many applications in Space Situation Awareness, for example in the design of disposal trajectories from Medium Earth Orbits (MEO) [1], or Highly Elliptical Orbits or Libration Point Orbits [2], or in the prediction of spacecraft re-entry, or in the modelling of the evolution of high area-to-mass ratio objects. On the other hand, the natural dynamics can be leveraged to reduce the propellant requirements, thus creating new opportunities. Orbit perturbations due to solar radiation pressure, atmospheric drag, third body effects, non-spherical gravity field, etc., play an important role.

The semi-analytical technique based on averaging is an elegant approach to analyse the effect of orbit perturbations. It separates the constant, short periodic and long-periodic terms of the disturbing function. The short-term effect of perturbations is eliminated by averaging the variational equations, or the corresponding potential, over one orbit revolution of the small body. Indeed, averaging corresponds to filtering the higher frequencies of the motion (periodic over one orbit revolution), which typically have small amplitudes. The resulting system allows a deeper understanding of the dynamics. Moreover, using the average dynamics reduces the computational time for numerical integration as the stiffness of the problem is reduced, while maintaining sufficient accuracy compatible with problem requirements also for long-term integrations.

This paper presents the Planetary Orbital Dynamics (PlanODyn) suite for long term propagation in perturbed environment. The Planetary Orbital Dynamics suite was developed within the FP7 EU framework in the Marie Skłodowska-Curie Actions[2]. It was originally designed for the analysis of Highly Elliptical Orbits disposals by enhancing the effect of natural perturbations [3] and it has been later extended to treat also Medium Earth Orbits [1] and Low Earth Orbits.

PlanODyn implements the orbital dynamics written in orbital elements by using semi-analytical averaging techniques [4]. The perturbed dynamics is propagated in the Earth-centred dynamics by means of the single and double averaged variation of the disturbing potential.

In this paper different application scenarios of PlanODyn are shown: the behaviour of quasi-frozen solutions appearing for high inclination and High Eccentricity Orbits (HEO) can be reproduced. In addition, to allow meeting specific mission constraints, stable conditions for quasi-frozen orbits can be selected as graveyard orbits for the end-of-life of HEO missions, such as XMM-Newton. On the opposite side, unstable conditions can be exploited to target an Earth re-entry; this is the case of the end-of-life of INTEGRAL mission, requiring a small delta-v manoeuvre for achieving a natural re-entry assisted by perturbations. Maps of stable and unstable HEOs are built, to be used as preliminary design tool for graveyard or frozen orbit design or natural re-entry trajectories at the end-of-life. Moreover, the application of PlanODyn to design end-of-life disposal from medium Earth orbits through passive solar sailing will be demonstrated. Two solar sail strategies are here analysed and compared. In Section 2 the dynamical models are explained and the basics

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[2] http://spacedebris-ecm.com/. Last retrieved 29/10/2015

of averaging. The two application of PlanODyn to the design of end-of-life of HEOs and the deorbiting through solar sailing are shown in Sections 4 and 5.

2. SEMI ANALYTICAL PROPAGATION BASED ON AVERAGED DYNAMICS

The orbit propagator within PlanODyn implements the single and double averaged dynamics of the Lagrange or Gauss planetary equations written in orbital elements (i.e. Keplerian or non-singular equinoctial elements depending on the application). It also allows the analytical estimation of the Jacobian matrix to be used for calculating the state transition matrix for sensitivity analysis, stability studies and uncertainty propagation.

PlanODyn propagates the Earth-centred dynamics by means of the averaged variation of the disturbing potential [4]. For the single average approach the averaging is performed over the orbit revolution of the spacecraft around the central body, for the double average approach the averaging is the also performed over the revolution of the perturbing body around the central body. Long period and secular effects are described in the current version, while an extension is foreseen to retrieve the short periodic effects. The perturbations implemented are: solar radiation pressure, atmospheric drag, zonal and tesseral harmonics of the Earth's gravity potential, third-body perturbations (i.e., the Sun and the Moon). In the case the effect of perturbations is conservative, this is described through a disturbing potential R which contains the terms due to the Earth gravity, the third body effect of the Sun and the Moon and Solar Radiation Pressure (SRP)

$$R = R_{\text{Earth gravity}} + R_{3-\text{Sun}} + R_{3-\text{Moon}} + R_{SRP}$$

In case of a conservative effect of the disturbance, the variation of the orbital elements is described through the planetary equations in the Lagrange form [5] that can be written in condensed form as

$$\frac{d\boldsymbol{\alpha}}{dt} = f\left(\boldsymbol{\alpha}, \frac{\partial R}{\partial \boldsymbol{\alpha}}\right)$$

where $\boldsymbol{\alpha}$ is here used as the vector of the Keplerian elements $\boldsymbol{\alpha} = \begin{bmatrix} a & e & i & \Omega & \omega & M \end{bmatrix}^T$, with a the semi-major axis, e the eccentricity, i the inclination, Ω the right ascension of the ascending node, ω the anomaly of the perigee and M the mean anomaly. Through the averaging technique, the potential can be replaced by the orbit-averaged form of the disturbing function

$$\bar{R} = \bar{R}_{\text{Earth gravity}} + \bar{R}_{3\text{B}-\text{Sun}} + \bar{R}_{3\text{B}-\text{Moon}} + \bar{R}_{SRP}$$

obtained under the assumption that the orbital elements are constant over one orbit revolution of the spacecraft around the central planet. Therefore, the variation of the mean elements is described by:

$$\frac{d\bar{\boldsymbol{\alpha}}}{dt} = f\left(\bar{\boldsymbol{\alpha}}, \frac{\partial \bar{R}}{\partial \boldsymbol{\alpha}}\right)$$

where now $\bar{\boldsymbol{\alpha}}$ is the vector of the mean orbital elements. In the case of the drag effect which is not conservative, the Gauss form of planetary equations are used instead.

3. DYNAMICS MODEL

In this section the dynamics model implemented in PlanODyn is summarised (Fig. 1). The averaged and double averaged equations are integrated with an explicit Runge-Kutta (4,5) method, the Dormand-Prince pair [6].

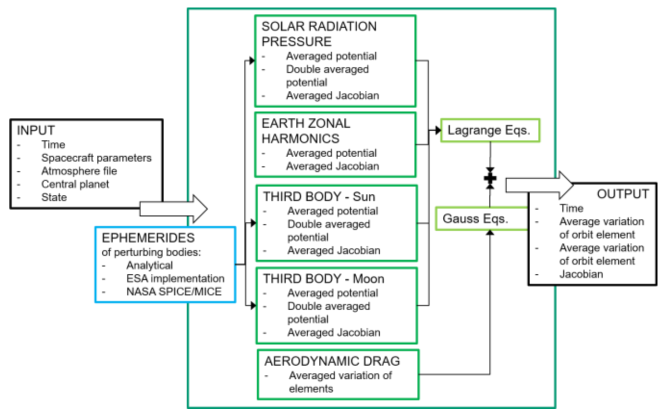

Fig. 1: PlanODyn dynamics schematics.

3.1. Ephemerides

The ephemerides of the Sun and the Moon and other perturbing bodies and the ephemerides of the Sun for the effect of SRP are predicted using three models that the user can select from:

1. An analytical approximation based on polynomial expansion in time (Ref. [7] for planets, [8] for Moon);
2. Numerical ephemerides from an ESA implementation;
3. Numerical ephemerides through the NASA SPICE/MICE toolkit[3]

3.2. Third body perturbation model in single and double averaged dynamics

The disturbing potential due to the third body perturbation is [9]:

[3] https://naif.jpl.nasa.gov/naif/toolkit.html. Last retrieved 29/10/2015.

$$R_{3\mathrm{B}}(r,r')=\frac{\mu'}{r'}\left(\left(1-2\frac{r}{r'}\cos\psi+\left(\frac{r}{r'}\right)^2\right)^{-1/2}-\frac{r}{r'}\cos\psi\right) \quad (1)$$

where μ' is the gravitational coefficient of the third body, r and r' are the magnitude of the position vector $\mathbf{r}$ and $\mathbf{r}'$ of the satellite and the third body with respect to the central planet, respectively, while their orientation is expressed by the angle ψ between $\mathbf{r}$ and $\mathbf{r}'$. The disturbing potential can be expressed as function of the spacecraft's orbital elements, choosing as angular variable the eccentric anomaly E, the ratio between the orbit semi-major axis and the distance to the third body r' on its mean circular orbit $\delta = a/r'$ and the orientation of the orbit eccentricity vector with respect to the third body [10]:

$$A=\hat{\mathbf{P}}\cdot\hat{\mathbf{r}}'$$

$$B=\hat{\mathbf{Q}}\cdot\hat{\mathbf{r}}'$$

where the eccentricity unit vector $\hat{\mathbf{P}}$, the semilatus rectum unit vector $\hat{\mathbf{Q}}$ and the unit vector to the third body $\hat{\mathbf{r}}'$ are expressed with respect to the equatorial inertial system, through the composition of rotations [4]. $A(\Omega,i,\omega,\Omega',i',\omega'+f')$ and $B(\Omega,i,\omega,\Omega',i',\omega'+f')$ can be expressed as function of the orbital elements [11] and the variables Ω', ω', i' and f', which are respectively the right ascension of the ascending node, the anomaly of the perigee, the inclination and the true anomaly of the perturbing body on its orbit (described with respect to the Earth-centred equatorial inertial system). Under the assumption that the parameter δ is small (i.e., the spacecraft is far enough from the perturbing body), Eq. (1) can be rewritten as a Taylor series in δ. Then, the average operation in mean anomaly can be performed, assuming that the orbital elements of the spacecraft a, e, i, Ω and ω are constant over one orbit revolution, to obtain the *single* averaged potential of the third body perturbation:

$$\bar{R}_{3\mathrm{B}}(r,r')=\frac{\mu'}{r'}\sum_{k=2}^{\infty}\delta^k\bar{F}_k(A,B,e) \quad (2)$$

Under the further assumption that the orbital elements do not change significantly during a full revolution of the perturbing body (e.g., Moon or Sun) around the central body (i.e., Earth), the variation of the orbit over time can be approximately described through the disturbing potential *double* averaged over one orbit evolution of the s/c and over one orbital revolution of the perturbing body around the Earth:

$$\bar{\bar{R}}_{3\mathrm{B}}(r,r')=\frac{\mu'}{r'}\sum_{k=2}^{\infty}\delta^k\bar{\bar{F}}_k(e,i,\Delta\Omega,\omega,i') \quad (3)$$

where $\Delta\Omega=\Omega-\Omega'$. In [4] we decided to express the double-averaged potential with respect to the Keplerian elements described in the Earth's centred equatorial reference system as this allows embedding the ephemerides of the Moon and the Sun, avoiding the simplification that Moon and Sun's orbit are on the same plane. The single averaged and the double averaged disturbing potential in Eqs. (2) and (3) can be used for predicting the orbit evolution by computing their particle derivatives with respect to the orbital elements and inserting them in the Lagrange equations [5] to compute the single (using Eq. (2)) and double (using Eq. (3)) variation of the orbital elements. It is important to remind that the modelling of the long-term orbit evolution through Eqs. (2) and (3) represents a good approximation as long as the parameter of the Taylor series $\delta = a/r'$ remain small enough. This is the case of Earth centred orbits (MEO, HEO up to the ones of INTEGRAL and XMM-Newton).

3.3. Solar radiation pressure

Solar radiation pressure is modelled through a cannonball model, where the characteristic acceleration

$$a_{\mathrm{SRP}}=\left(p_{\mathrm{SR}}c_R A_{\odot}\right)/m$$

is expressed as function of the solar pressure $p_{\mathrm{SR}}=4.56\times10^{-6}\,N/\mathrm{m}^2$ (at 1 AU), c_{R} the reflectivity coefficient which measures the momentum exchange between incoming radiation and the spacecraft and the cross area of the spacecraft over its mass $A_{\odot}/m$. Eclipses are not considered at this stage but they can be easily be included in the semi-analytical method by the approach used in [12]. The averaged potential due to SRP can be written as [13]:

$$\begin{aligned}\bar{R}_{SRP}=Cna^2e\big(&\cos\omega\left(\cos\Omega\cos\lambda_{\mathrm{Sun}}+\sin\Omega\sin\lambda_{\mathrm{Sun}}\cos\varepsilon\right)+\\ &\sin\omega\big(\cos\Omega\cos i\sin\lambda_{\mathrm{Sun}}\cos\varepsilon+\\ &+\sin i\sin\lambda_{\mathrm{Sun}}\sin\varepsilon-\sin\Omega\cos i\cos\lambda_{\mathrm{Sun}}\big)\big)\end{aligned}$$

where λ_{Sun} is the longitude of the Sun on the ecliptic and ε is the obliquity angle between the equator plane and the ecliptic plane, and the solar radiation pressure parameter is:

$$C=\frac{3}{2}a_{\mathrm{SRP}}\frac{a^2}{\mu_{\mathrm{Earth}}}n$$

with n is the orbit angular velocity and μ_{Earth} the gravitational constant of the Earth.

3.4. Earth gravity

For many applications (HEO, MEO) the important term for the Earth gravity is the effect of the Earth's oblateness J_2.

$$\bar{R}_{J_2}=W\frac{na^2}{6}\frac{3\cos^2 i-1}{\left(1-e^2\right)^{3/2}}$$

where the oblateness parameter W collects is defined as

$$W = \frac{3}{2} J_2 \frac{R_{\text{Earth}}^2}{a^2} n$$

$J_2 = 1.083 \cdot 10^{-3}$ denotes the second zonal harmonic coefficient and R_{Earth} is the mean radius of the Earth. In PlanODyn the averaged potential of the Earth gravity harmonics is implemented up to order 10 while a recursive approach is under development. The tesseral harmonics can be also included. For the full gravity field instead two options are available to the user through the use of the Kaula and the Cunningham algorithm.

3.5. Aerodynamics drag

The secular disturbing effect on the orbit due to atmospheric drag can be also modelled with semi-analytical techniques. However, atmospheric drag is the only non-conservative force, in this case rather than using the disturbing potential and Lagrange planetary equations, the Gauss form of the equations is used and averaged to obtain the secular and long-term effect of drag. The semi-analytical approximation for drag is taken from King-Hele [14] that gives a set of semi-analytical expression depending on the drag regimes (eccentricity regimes, area-to-mass regimes). These equations are based on the assumption of a time-independent, spherically-symmetric atmosphere with a density that varies exponentially with altitude *h*, according to an exponential law. A piece-wise exponential model is currently implemented in PlanODyn [8]. As long as a fitting table is provided, the user can select among any density model.

3.6. Validation

PlanODyn was successfully validated against the actual ephemerides of INTEGRAL from NASA HORIZONS available over the period 2002/10/18 to 2011/12/30 and the actual ephemerides of XMM-Newton, given by ESA, the long-term propagation of high LEO under the effects of solar radiation pressure and drag, MEO orbits considering only SRP and Earth's oblateness. Current work is being devoted to validate the model of luni-solar perturbation in MEO. Therefore, PlanODyn can be used to accurately access the orbit evolution around the Earth, to be then refined with a full perturbation model.

4. END-OF-LIFE DISPOSAL FROM HIGLY ELLIPTICAL ORBIT EXPLOITING THIRD BODY PERTURBATION

The double averaged dynamics under the effects of luni-solar perturbation and Earth's oblateness were used to study the long-term evolution of HEOs [3]. This analysis was aimed at characterising the orbit evolution in the phase space to identify the stability conditions within a wide set of initial conditions. Maps of maximum eccentricity variations were built as a measure of the orbit stability. A grid was built in the domain of inclination, eccentricity and anomaly of the perigee. Note that, inclination and anomaly of the perigee are described with respect to the Moon plane reference system. Each initial condition of the grid was propagated backward and forward in time over with PlanODyn. For each initial condition the change between the minimum and the maximum eccentricity attained during the period analysed is stored.

Fig. 2 shows the Δe map for a semi-major axis of 87,736 km. Note that all initial conditions are integrated with the same stating date, which corresponds to a given initial condition of the Sun and the Moon with respect to the Earth. A different starting date would give a different net change of eccentricity for each initial condition in the plane but would not change the characteristics of the solutions. We can recognise the existence of a quasi-frozen solution at 180 degrees and high eccentricity.

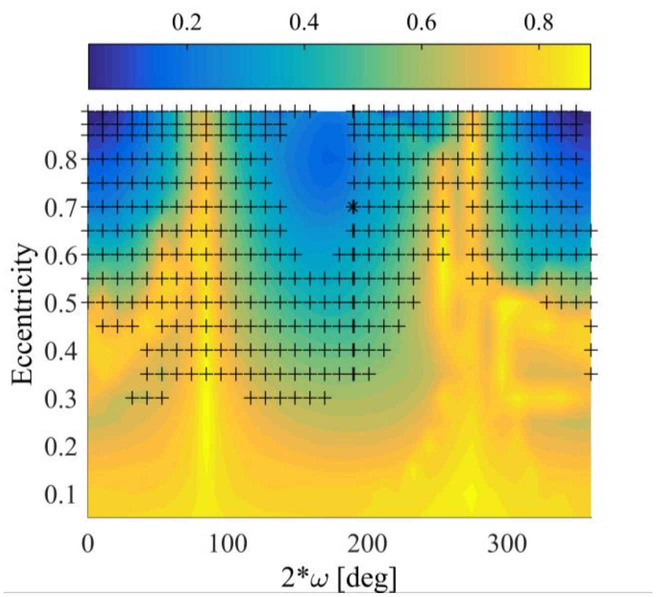

Fig. 2: Luni-solar + zonal Δ*e* maps for semi-major axis of 87736 km and initial inclination with respect to the orbiting plane of the Moon of 64.2 degrees (INTEGRAL-like orbit).

By exploiting the finding of these maps, perturbation-enhanced transfer can be designed to achieve re-entry targeting trajectories in the phase space with a high variation of Δe such the spacecraft eccentricity is increased (at constant semi-major axis) up to the critical eccentricity for Earth e-entry: $e_{\text{crit}} = 1 - \left(R_{\text{Earth}} + h_{p,\,\text{drag}}\right)/a$ where $h_{p,\,\text{drag}}$ is the target orbit perigee, which needs to be selected well inside the Earth atmosphere to ensure a safe re-entry [3, 15]. On the opposite side, perturbation-enhanced transfer into a graveyard orbit can be obtained by targeting low variation of Δe in the $e-\omega$ phase space (with respect to the perturbing body) as this represents quasi-stable orbits.

In the framework of disposal design, the double average dynamics in PlanODyn was used to explain the evolution of the orbital elements using simplified models (i.e. considering only the Moon and the effect of the Earth's oblateness). In this way the long-term oscillation of the eccentricity due to third body perturbations were identified and a preliminary strategy for perturbation enhanced deorbiting defined. The single averaged algorithm was instead used within a global optimisation method to search, for a given starting date, the optimal manoeuvre to achieve deorbiting. In order to perform a parametric analysis on the starting date for the disposal manoeuvre, many initial condition for considering an impulsive change in velocity were identified during the natural orbit evolution. Then, the time instant corresponding to the minimum required manoeuvre was selected. Fig. 3 show as example the optimised re-entry disposal trajectory for the INTEGRAL mission [3].

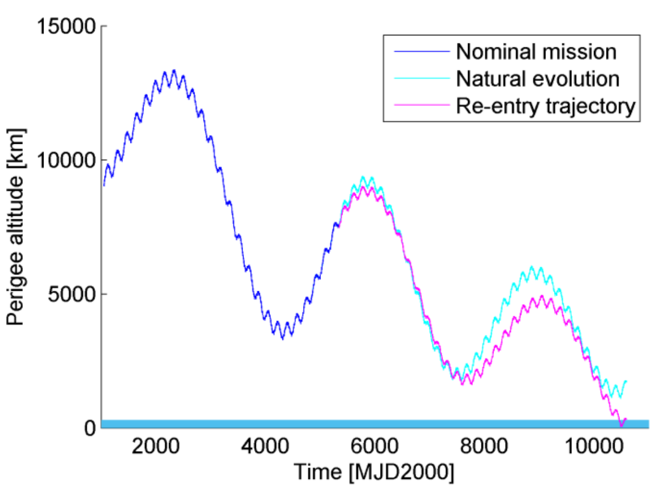

Fig. 3. End-of life INTEGRAL disposal [3].

5. PASSIVE SOLAR SAILING FOR END-OF-LIFE DISPOSAL

The increase of space debris in the past decades has boosted the need of end-of-life disposal devices for future and orbiting spacecraft. Among the passive deorbiting methods, solar and drag sailing have been analysed and technology demonstrator are under development. One application of PlanODyn focused on the design of disposal using a solar sail or a deployable reflective surface to increase the area-to-mass of the spacecraft and enhance the effects of solar radiation pressure and atmospheric drag.

While the effect of drag can be enhanced by maximising the cross area to the velocity vector, the effect of solar radiation pressure can be exploited in two different ways. Conventional active solar sailing for deorbiting aims at maximising the cross area of the sail perpendicular to the spacecraft-Sun direction when the spacecraft is moving towards the Sun, while the sail area is minimised when the spacecraft is flying away from the Sun. In this way the semi-major axis and thus the energy of the orbit is continuously decreased.

A novel strategy, proposed by Lücking et al. [16], keeps the reflective surface always oriented towards the Sun and control it though passive stabilisation (this is achieved through a pyramidal shape sail). Deorbit in this case is achieved by increasing the eccentricity of the orbit, at a quasi-constant semi-major axis, until the critical eccentricity for re-entry is reached. PlanODyn was used coupled with a Newton-Rapson method to define the sail requirements for deorbiting. This can be defined by computing the minimum effective area-to-mass $\left(c_R A_{\odot}\right)/m$ that allows the eccentricity to grow up to the critical value (eccentricity value such as the orbit perigee is below 200 km). The effective are-to-mass requirement for an initial circular orbit is shown in Fig. 4 for various inclination and semi-major axis.

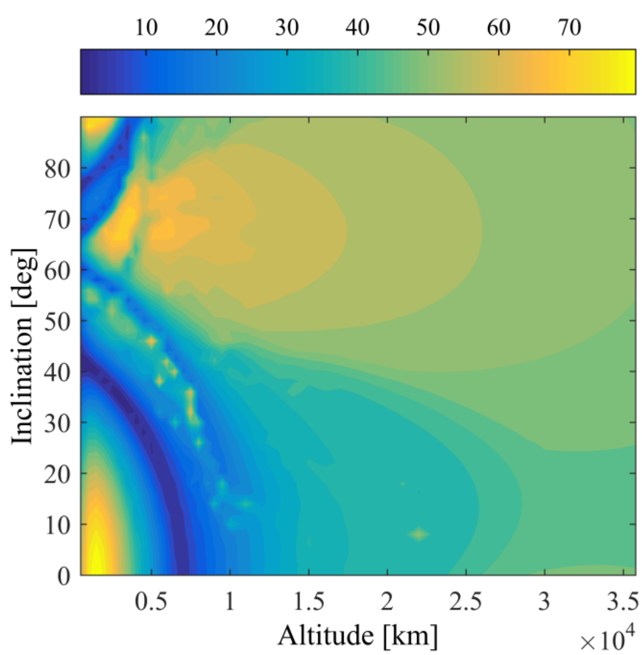

Fig. 4: Area-to-mass times reflectivity coefficient [m^2/kg] to de-orbit from circular orbit and Ω_0 = 0 degrees with sail passive mode strategy.

The sail passive mode offers a deorbiting possibility within 5 years, and a sail parameter of less than 6 m^2/kg, down to 0.75

m^2/kg (42 degrees inclination). Importantly, it extends the range of solar sailing deorbiting up to 7000 km altitude (for low inclination), down to 500 km (for 42 degrees inclination). The deorbiting time is always below 250 days (Fig. 5), apart of an area of the domain where the J_2 and SRP dynamics has a bifurcating behaviour [16].

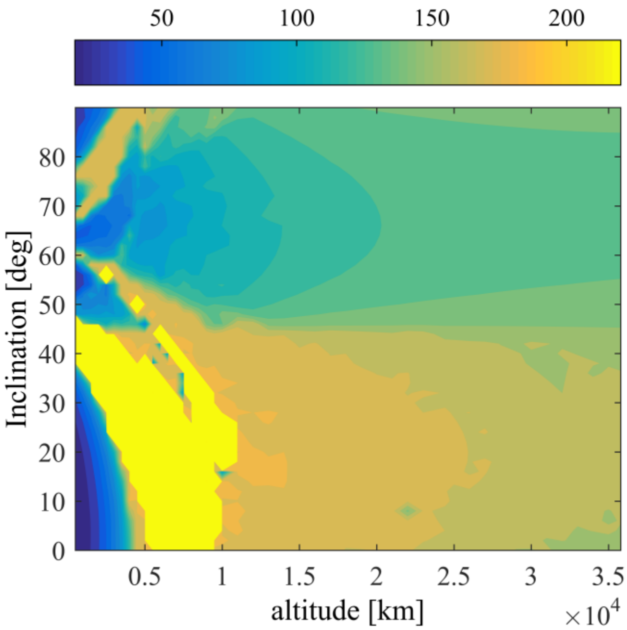

Fig. 5: Time to de-orbit in days from circular orbit and Ω_0 = 0 degrees with sail passive mode strategy.

In the simple area-augmented passive deorbiting the effect of solar radiation pressure is exploited by artificially increasing the area-to-mass ratio of the spacecraft. An alternative strategy was also proposed that modulates the effect of solar radiation pressure during deorbiting as a function of the Sun-perigee angle. The effect of solar radiation pressure is exploited only when the secular and long-term evolution of the eccentricity is positive, while the area-to-mass increasing device is de-activated, otherwise. In this way, a lower area-to-mass is required to reach the critical eccentricity, as more than one cycle in the phase space are allowed (Fig. 6). The number of cycles is strictly fixed by the maximum time allowed for deorbiting and determines also the number of time the area-to-mass increasing device needs to be activated/deactivated. Such an effect can be achieved by changing the attitude of a solar sail with respect to the Sun on an average of 6 months, or by designing a reflective surface with a pyramidal shape, whose area can be controlled.

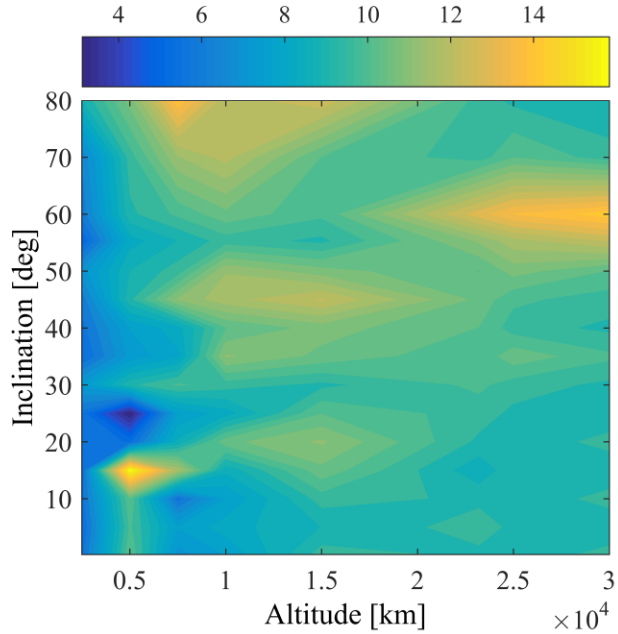

Fig. 6: Area-to-mass times reflectivity coefficient [m^2/kg] to de-orbit from circular orbit and Ω_0 = 0 degrees with sail passive modulation mode strategy.

6. CONCLUSION

This article describes the Planetary Orbital Dynamics (PlanODyn) suite for long term propagation in perturbed environment. The dynamical model used for deriving the averaged equation of motion was summarised and the physical models for the spacecraft and planetary environment presented. As the example application show averaging techniques can be used to highlight the dynamical properties of the region around the Earth. This insights have pave the way o the identification of deorbiting strategies exploiting the effects of orbit perturbations. The design of the re-entry of the INTEGRAL mission was performed and the requirement for solar sail deorbiting can be found. Different disposal strategies can be identified and compared. PlanODyn have been also applied to study the re-entry from Geostationary Transfer Orbits, to propagate clouds of debris fragments and swarms of high area-to-mass spacecraft. Current effort is exploiting average dynamics for uncertainty estimation.

7. AKNOLEDGMENTS

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